Exam. Code : 211002
Subject Code : 4976

M.Sc. (Mathematics) 2nd Semester
TENSORS AND DIFFERENTIAL GEOMETRY
Paper—MATH-562

Time Allowed—2 Hours] [Maximum Marks—100

Note :— There are ***eight*** questions of equal marks. Candidates are required to attempt any ***four*** questions.

1. (a) Define Contraction. State and prove contraction theorem.

   (b) Define Kronecker delta and prove that it is a tensor of order two.

2. (a) Show that there exists no distinction between contravariant and covariant vectors when we restrict ourselves to rectangular Cartesian transformation of coordinates.

   (b) Show that the covariant derivatives of the tensors g_{ij}, g^{ij} and σ^i_j all vanish identically.

3. (a) Define Principal normal and Binormal. Also state and prove Serret-Frenet formulae.

   (b) Show that the principal normals at consecutive points do not intersect unless $\tau = 0$.

4. (a) Define Helix. Show that the necessary and sufficient condition for a curve to be a helix is that its curvature and torsion are in constant ratio.

   (b) Prove that the corresponding points on the spherical indicatrix of the tangent to a curve C and on the indicatrix of the binormal to C have parallel tangent line.

5. (a) Find the envelope and the edge of regression of the spheres which pass through a fixed point and whose centres lie on a given curve.

   (b) Find the fundamental magnitudes and normal to the surface.

6. (a) Show that the parametric curves on the surface are asymptotic iff $L = 0$, $N = 0$, $M \neq 0$.

   (b) State and prove theorem of Beltrami and Ennper.

7. (a) Show that a curve on a sphere is a geodesic iff it is a great circle.

   (b) Find the expression for torsion of a geodesic.

8. (a) State and prove Gauss-Bonnet theorem.

   (b) State and prove Tissot's theorem.